5 D-Plane Symmetric Solutions in $f(R,T)$ Gravity with Quadratic EoS

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Abstract: In this paper, we investigate a five-dimensional plane-symmetric Solutions within the framework of f(R,T) gravity by assuming a quadratic equation of state (EoS). The physical and geometrical characteristics of the universe model have been also studied.

Keywords- 5D-Plane symmetric space-time; f(R,T) theory of gravity; Perfect fluid; Quadratic equation of state

Introduction:

A key component of contemporary astrophysics and cosmology is the study of cosmological models, which offer a theoretical framework for comprehending the universe's large-scale structure and development. Because they are homogeneous yet anisotropic, plane-symmetric cosmological models in particular have garnered interest as a means of characterizing the early cosmos and investigating potential departures from isotropy. The cosmos is expanding at an accelerated rate, which cannot be adequately explained by general relativity alone, according to mounting observational data in recent years. In order to explain the observed acceleration and obtain a better understanding of the dynamical behavior of the universe, this has prompted a great deal of study on modified theories of gravity as well as the use of extended cosmological models.

Numerous observational findings over the past few decades have demonstrated that the cosmos is expanding more quickly. This finding is substantially supported by independent evidence from the PLANK team [11], baryon acoustic oscillations [8–10], cosmic microwave background anisotropies [7], and Type-Ia supernovae experiments [1–6]. One of the most difficult issues in modern cosmology is still how to explain this rapid expansion. Modifications to the general theory of relativity itself or the addition of an exotic dark energy component with negative pressure are the two main strategies that have been put forth.

Harko et al. [12] suggested the f(R,T) theory of gravity, which is an extension of general relativity in which the gravitational action relies on both the trace T of the energy–momentum tensor and the Ricci scalar R. Plane-symmetric cosmological models are among the many cosmological models that have been thoroughly studied using this modified theory.

In this context, we investigate the five dimensional plane symmetric solutions in the presence of a quadratic equation of state with a cosmological parameter in the f(R, T) theory of gravity. The quadratic equation of state is a more general form of the equation of state, which can describe a wide range of cosmic fluids, including dark energy and dark matter.

By taking domain walls into account, Biswal et al. [13] examined a five-dimensional Kaluza–Klein cosmological model in the context of f(R,T) gravity and found exact solutions using Berman's special law of variation of the Hubble parameter, which results in a constant deceleration parameter. In the framework of f(R,T) gravity, Dasunaidu et al. [14] studied non-static five-dimensional spherically symmetric cosmological models with heavy strings. In the context of modified gravity theory, Pawar et al. [15] examined a Bianchi type-V cosmological model with a perfect fluid and heat conduction.

$f(R,T)$ Gravitation Theory

The action for the modified $f(R,T)$ gravity

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}d^5x + \int L_m\sqrt{-g}d^5x \tag{1}$$

Where R is the Ricci scalar, T is the trace of energy momentum tensor and $f(R,T)$ is an arbitrary function of R and T . L_m is the matter Lagrangian density.

Varying the action S with respect to the metric tensor g_{ij}, the field equations in $f(R,T)$ theory of gravity are given by

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i\nabla_j - g_{ij})f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{2}$$

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Where $\Box \equiv \nabla^i \nabla_i$ is the De Alembert's operator, f_R and f_T are the ordinary derivatives with respect to Ricci scalar R and trace of energy momentum tensor T respectively.

Contraction of equation (2) gives

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - Tf_T(R,T) - f_T(R,T)\Theta \quad (3)$$

Where $\Theta = g^{ij}\Theta_{ij} = \Theta_i^i$.

The stress-energy tensor of the matter Lagrangian is derived as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \quad (4)$$

The perfect fluids described by an energy density ρ and pressure p. The four velocity $u^i = (0,0,0,1)$ in co-moving co-ordinates and there is no any unique definition of matter Lagrangian. Thus we can assume

$L_m = -p$, which gives

$$\Theta_{ij} = -2T_{ij} - p\,g_{ij}, \quad (5)$$

Using equations (5), the field equations (2) can be written as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} - (\nabla_i \nabla_j - g_{ij}\Box)f_R(R,T) = 8\pi T_{ij} + f_T(R,T)T_{ij} + pf_T(R,T)g_{ij} \quad (6)$$

By using the field equations for $f(R,T)$ gravity for the choice of the functional

$f(R,T) = R + 2f(T)$.

Where $f(T)$ is an arbitrary function of stress energy tensor of matter.

Here we consider $f(T) = \mu T$ and $f'(T) = {df}/{dT}$ where μ is a constant.

We obtain the field equations (6) can be expressed as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [f(T) + 2p\,f'(T)]g_{ij}, \quad (7)$$

Metric and the field equations

We consider plane symmetric space-time is

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2dz^2 - C^2du^2 \quad (8)$$

Where A, B and C cosmic scale factors are is function of t.

By using, the field equations (7) with the help of equation (4) for the metric

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(8) Can be explicitly written as

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}=8\pi p+4p\mu-\mu\rho \tag{9}$$

$$\frac{2\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{C}}{AC}=8\pi p+4p\mu-\mu\rho \tag{10}$$

$$\frac{2\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+\frac{2\dot{A}\dot{B}}{AB}=8\pi p+4p\mu-\mu\rho \tag{11}$$

$$\frac{2\dot{A}\dot{B}}{AB}+\frac{2\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}^2}{A^2}=8\pi\rho+\rho\mu+2\mu p \tag{12}$$

Where overhead (•) denotes derivative with respect to time t.

The average scale factor & spatial volume of the universe is defined as

$$a^4=V \qquad V=A^2BC \tag{10}$$

The generalized mean Hubble's parameter is given as

$$H=\frac{1}{4}\left[\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right] \tag{11}$$

Where $H_x=H_y=\frac{\dot{A}}{A}$, $H_z=\frac{\dot{B}}{B}$ and $H_u=\frac{\dot{C}}{C}$ are the directional Hubble's parameter in the x. y z and u axes direction respectively.

The expansion scalar is given by

$$\theta=\left[\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right] \tag{12}$$

The shear scalar and the mean anisotropic parameter is define as

$$\sigma^2=\frac{4}{2}A_mH^2 \tag{13}$$

and

$$A_m = \frac{1}{4H^2}\left[\sum_{i=1}^{3} H_i^2 - 4H^2\right] \quad (14)$$

The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \quad (15)$$

Solutions of the field Equations:

Subtract Equations (10) from Equations (11)

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0 \quad (16)$$

Integrate Equation (18)

$$C = Bk \quad (17)$$

Where k is integration constant. Without loss of generality.

We assume that $k = 1$, so that we have

$$C = B \quad (18)$$

Inserting Eqns (9)-(12), we get

$$\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 8\pi p + 4p\mu - \mu\rho \quad (19)$$

$$\frac{2\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} = 8\pi p + 4p\mu - \mu\rho \quad (20)$$

$$\frac{4\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}^2}{A^2} = 8\pi\rho + \rho\mu + 2\mu p \quad (21)$$

We assume that shear scalar σ is proportional to the expansion scalar θ which gives a linear relationship between the scale factors A & B as fallows

$$B = A^n \quad (22)$$

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Where $n \neq 1$ is an arbitrary constant. This constant gives that model becomes anisotropic. Otherwise, if $n = 1$ model becomes an isotropic model.

Subtract equation (19) from equation (20) and by using equation (22), we get equation can be written as

$$\frac{\ddot{A}}{\dot{A}} = (1-n)\frac{\dot{A}}{A} \tag{23}$$

Solving equation (23) with the help of equation (22), we get values of scale factor A B & C is given as

$$A = (nc_1 t + c_2)^{1/n}\ ,\ B = (nc_1 t + c_2) \qquad C = (nc_1 t + c_2) \tag{24}$$

Quadratic EoS Solutions

The quadratic EoS has become important in the cosmological models to observe the dark energy and dynamics of GR. Hence, we are motivated to assume the three different type of pressure and energy density of the fluid satisfying quadratic EoS which can be expressed in the form

$$p = \alpha\rho^2 - \rho \tag{25}$$

In this case, by using equation (24) in equation (19)-(21) with help of equation (25)

We can obtain energy density, pressure of the fluid and $f(R,T)$ gravity as

$$\rho = \frac{c_1}{(nc_1 t + c_2)}\sqrt{\frac{1-2n}{(4\pi+\mu)\alpha}} \tag{26}$$

$$p = \frac{c_1}{(nc_1 t + c_2)}\sqrt{\frac{1-2n}{(4\pi+\mu)}}\left(\frac{c_1}{(nc_1 t + c_2)}\sqrt{\frac{1-2n}{(4\pi+\mu)}} - \frac{1}{\sqrt{\alpha}}\right) \tag{27}$$

$$f(R,T) = \frac{6c_1^2}{(nc_1 t + c_2)^2}\left[\frac{2(2\pi+\mu n)}{4\pi+\mu}\right] + \frac{4\pi c_1}{(nc_1 t + c_2)}\sqrt{\frac{1-2n}{(4\pi+\mu)\alpha}} \tag{28}$$

Some kinematical properties of the model

The average scale factor & spatial volume of the universe is obtained as

$$a=(nc_1t+c_2)^{\frac{1+n}{2n}} \quad \text{and} \qquad V=(nc_1t+c_2)^{\frac{2+2n}{n}} \tag{29}$$

The generalized mean Hubble's parameter is found to be

$$H=\frac{c_1(n+2)}{4(nc_1t+c_2)} \tag{30}$$

The expansion scalar of model is

$$\theta=\frac{c_1(n+2)}{(nc_1t+c_2)} \tag{31}$$

The shear scalar and the mean anisotropic parameter of the model is given as

$$\sigma^2=\frac{(n^2-4n+1)c_1^2}{3(nc_1t+c_2)^2} \tag{32}$$

$$A_m=\frac{4(n^2-4n+1)}{(n+2)^2} \tag{33}$$

The deceleration parameter of the model is

$$q=-\frac{4(1-n)}{(2+n)} \tag{34}$$

CONCLUSION

In this paper we have observed, five dimensional plane symmetric cosmological model with quadratic Eos in $f(R,T)$ theory of gravity assuming the quadratic equation of state which can be expressed in the form $p=\alpha\rho^2-\rho$, we obtain the energy density, pressure for $f(R,T)$ gravity using $f(R,T)=R+2f(T)$ in three cases. Some kinematical properties of the model have been studied.

- For $n>1$, this show that the deceleration parameter becomes negative and universe is in accelerated expansion.
- The average scale factor and spatial volume of the universe becomes constant taking for $t\rightarrow 0$,

- If $t \to \infty$, then generalized mean Hubble parameter, expansion scalar and shear scalar are zero.
- For $n > 1$, the value of Hubble parameter remains positive.

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